

# Large-scale dynamical simulations of the three-dimensional $XY$ spin glass

Qing-Hu Chen\*

*Center for Statistical and Theoretical Condensed Matter Physics, Zhejiang Normal University,  
Jinhua 321004, People's Republic of China*

*and Department of Physics, Zhejiang University, Hangzhou 310027, People's Republic of China*

(Received 24 July 2009; revised manuscript received 30 August 2009; published 21 October 2009)

Large-scale simulations have been performed in the current-driven three-dimensional  $XY$  spin glass with resistively shunted-junction dynamics for sample sizes up to  $64^3$ . It is observed that the linear resistivity at low temperatures tends to zero, providing a strong evidence of a finite-temperature phase-coherence (i.e., spin-glass) transition. Dynamical scaling analysis demonstrates that a perfect collapse of current-voltage data can be achieved. The obtained critical exponents agree with those in equilibrium Monte Carlo simulations, and are compatible with those observed in various experiments on high- $T_c$  cuprate superconductors. It is suggested that the spin and the chirality order simultaneously. A genuine continuous depinning transition is found at zero temperature. For low temperature creep motion, critical exponents are evaluated, and a non-Arrhenius creep motion is observed in the low temperature ordered phase. It is proposed that the  $XY$  spin glass gives an effective description of the transport properties in high- $T_c$  superconductors with  $d$ -wave symmetry.

DOI: 10.1103/PhysRevB.80.144420

PACS number(s): 05.10.-a, 68.35.Rh, 74.25.-q

## I. INTRODUCTION

The ordering of the three-dimensional (3D)  $XY$  spin-glass model<sup>1</sup> has attracted considerable attention. Theoretically, earlier work suggests transitions at zero or very low temperatures. Following the pioneering work by Villain<sup>2</sup> where the role of chirality (i.e., vorticity) arising for noncollinear  $XY$  system was emphasized, Kawamura and Tanemura proposed a chiral-glass transition, even though the finite-temperature spin-glass transition vanishes.<sup>3</sup> Later on, by calculating the Binder ratio and the spin-overlap distribution function for lattice sizes up to  $L=16$ , Kawamura and Li have given a numerical evidence that the low temperature phase is a chiral glass without the conventional spin-glass order.<sup>4</sup>

However, this spin-chirality decoupling scenario is not consistent with several studies.<sup>5–10</sup> Maucourt and Gempel<sup>5</sup> and subsequently, Akino and Kosterlitz<sup>6</sup> found evidence for finite-temperature spin-glass transition from zero-temperature domain wall calculations. By performing a finite-size scaling analysis of the correlation length  $\xi$ , the most successful technique developed in the Ising spin glass,<sup>11</sup> Lee and Young<sup>8</sup> observed a transition at the same temperature for both spins and chiralities for lattice sizes up to  $L=12$ . Besides the equilibrium simulation, the resistive behavior in the  $XY$  spin glass has been also studied. In a vortex representation, Wengle and Young showed evidence of a resistive transition at finite temperatures<sup>12</sup> for lattice sizes up to  $L=10$ . Granato observed that the current-voltage characteristic in the  $XY$  spin glass with both bimodal and Gaussian couplings for the lattice size  $L=12$  exhibited scaling behavior,<sup>9,10</sup> which was interpreted in terms of both the phase-coherence (spin-glass) transition and the chiral-glass transition.

The controversy on the spin-chirality decoupling scenario is, however, still ongoing.<sup>13</sup> For the Heisenberg spin glass, the transitions at the same temperature was also observed for both spins and chiralities in Ref. 8 with the use of the same method for  $XY$  spin glass. However, subsequently, it is found

that the situation becomes rather unclear when results of larger sizes are included.<sup>14–18</sup> The data at the low temperatures in larger samples show rather marginal behavior, i.e., the system is close to the lower critical dimension. Recently, there seems a intensely competition for the lattice sizes accessible,<sup>16–18</sup> the record until now is  $L=48$ .<sup>18</sup> Motivated from the debate for Heisenberg case, Pixley and Young further studied the  $XY$  spin glass with larger lattice sizes up to  $L=24$ ,<sup>19</sup> and observed similar marginal behavior for low temperatures and large systems. So, it is also desirable to perform the simulations on very large sample for  $XY$  spin glass. In contrast with the Ising and Heisenberg spin glass, there is an additional technique in  $XY$  spin glass, that is the measurement of the resistivity from dynamical simulations.<sup>9,10,12</sup> Moreover, one can access larger system in this kind of dynamical simulations, because a steady state is more easily reached than equilibration due to the presence of the external driven force. It is also of interest to know whether the previous picture based on small systems ( $L \leq 12$ ) in the dynamical simulation<sup>9,10,12</sup> is modified on larger systems for the  $XY$  case.

On the one hand, the  $XY$  spin glass has found experimental realization, not only in layered manganite, e.g.,  $\text{Eu}_{0.5}\text{Sr}_{1.5}\text{MnO}_4$ ,<sup>20</sup> but also in high- $T_c$  cuprate superconductors with the  $d$ -wave pairing symmetry,<sup>21</sup> which can be regarded as a random distribution of  $\pi$  junctions.<sup>22–26</sup> The nature of  $d$ -wave symmetry will changes the sign of the coupling between  $XY$  spins, while the spin angle denotes the phase of the superconducting order parameters. So the  $XY$  spin glass is expected to be used to interpret the phenomena observed in high- $T_c$  cuprate superconductors, such as vortex glass phase,<sup>27–29</sup> a true superconducting state with vanishing linear resistivity. The evidences to support the existence of this phase have been reported in many experiments.<sup>30,31</sup>

Since the chiral variable in the  $XY$  spin glass can be defined as the vortex in the plaquette,<sup>9,10</sup> some techniques developed in superconducting vortex models<sup>32–34</sup> can be in turn employed to characterize the nature of the low temperature ordered state in the  $XY$  spin glass. As is well-known, in the

random pinning environment, the energy landscape for the vortex motion is highly nontrivial. The theoretical understanding for the nonlinear dynamics response has been advanced in many years.<sup>35</sup> But the full theoretical study is still very challenging, computer simulations are hopeful to provide useful insights.

In this paper, by resistively shunted-junction dynamics, we perform large-scale dynamical simulations in the 3D spin glass, both the phase-coherence transition temperature  $T_g$  and the critical exponents are estimated. The depinning transition at zero-temperature and creep motion below  $T_g$  is also investigated. The rest of the paper is organized as follows. Section II describes the model and dynamic method. Section III presents our main results, where some discussions are also performed. Finally, a short summary is given in the last section.

## II. MODEL AND DYNAMIC METHOD

The Hamiltonian of the 3D XY spin glass in the phase representation is given by<sup>8</sup>

$$H = - \sum_{\langle ij \rangle} J_{ij} \cos(\phi_i - \phi_j), \quad (1)$$

where the sum is over all nearest-neighbor pairs on a 3D square lattice,  $\phi_i$  specifies the phase of the superconducting order parameter on grain  $i$ ,  $J_{ij}$  denotes the strength of Josephson coupling between neighboring grains with zero mean and standard deviation unit. The present simulations are performed with the lattice size  $L=64$  for all directions, considerably larger than those in literature.

The resistivity-shunted-junction dynamics is incorporated in simulations. In this case, the current through a nearest-neighbor link (or bond) from grain  $i$  to grain  $j$  contains three contributions,

$$I_{ij} = I_{ij}^{(s)} + I_{ij}^{(n)} + \eta_{ij},$$

where  $I_{ij}^{(s)} = (2eJ_{ij}/\hbar)\sin(\phi_i - \phi_j)$  is the Josephson supercurrent,  $I_{ij}^{(n)} = \sigma(V_i - V_j)$  is the normal current with  $\sigma$  the conductance of the resistive shunt, and  $\eta_{ij}$  is a thermal noise current, independent from bond to bond, with zero mean and a correlator  $\langle \eta_{ij}(t) \eta_{ij'}(t') \rangle = 2\sigma k_B T \delta(t - t') \delta_{ij, i'j'}$ . The voltage on node  $i$  is given by  $V_i = (\hbar/2e)d\phi_i/dt$ . So the dynamical equation can be described as

$$\frac{\sigma \hbar}{2e} \sum_j (\dot{\phi}_i - \dot{\phi}_j) = - \frac{\partial H}{\partial \phi_i} + J_{\text{ext}, i} - \sum_j \eta_{ij}. \quad (2)$$

Here,  $J_{\text{ext}, i}$  is the external current, which is only applied to the boundary site  $i$  and vanishes inside the sample. In the following, the units are taken of  $2e = \hbar = \sigma = k_B = 1$ .

In the present simulations, a uniform external current  $I_x$  along  $x$  direction is fed into the system, the fluctuating twist boundary condition<sup>34</sup> is applied in the  $xy$  plane, and the periodic boundary condition is used in the  $z$  axis. In the  $xy$  plane, the supercurrent between sites  $i$  and  $j$  is now given by  $I_{i \rightarrow j}^{(s)} = J_{ij} \sin(\theta_i - \theta_j - A_{ij} - \mathbf{r}_{ij} \cdot \Delta)$ , with  $\Delta = (\Delta_x, \Delta_y)$  the fluctuating twist variable and  $\theta_i = \phi_i + \mathbf{r}_i \cdot \Delta$ . Since the new phase angle  $\theta_i$  is periodic, we obtain the voltage difference across the sample  $V_\alpha = -L\dot{\Delta}_\alpha$ ,  $\alpha = x, y$ . To achieve a given current flow  $I_\alpha$  through the sample, we require,

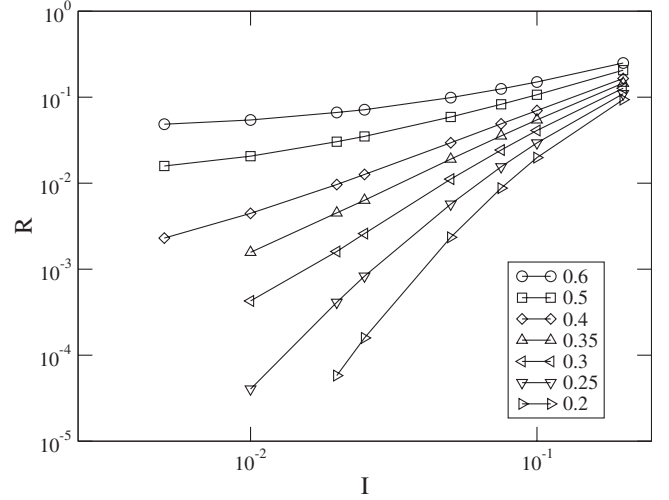


FIG. 1. Current-resistivity curves at various temperatures

$$\dot{\Delta}_\alpha = \frac{1}{L^3} \sum_{\langle ij \rangle_\alpha} [I_{i \rightarrow j}^{(s)} + \eta_{ij}] - I_\alpha, \alpha = x, y, \quad (3)$$

i.e., the sum of normal, super, and noise currents yields the desired total current. Equation (3) defines the dynamics for  $\Delta_\alpha$ .

The above equations can be solved efficiently by a pseudospectral algorithm<sup>32</sup> due to the periodicity of phase in all directions. The time stepping is done using a second-order Runge-Kutta scheme with  $\Delta t = 0.05$ . The time-averaged voltages are calculated over a long-time scale after reaching the steady state. To determine the steady state, we have checked  $\langle V \rangle_n$  for every  $(2^n - 2^{n-1})/\Delta t$  time steps. We assume that the system reaches a steady state when the fluctuation of the mean voltage  $|\langle V \rangle_n - \langle V \rangle_{n-1}|/\langle V \rangle_n$  is less than 0.5% for several  $n$ 's. Once this criterion is satisfied, we record the  $\langle V \rangle_n$  as the final estimate of the voltage. The value of the maximum  $n$  is typically 22 in the present simulations. Our results are based on one realization of disorder. The present system size is much larger than those reported in literature, a good self-averaging effect is expected. We have performed an additional simulations with a different realization of disorder for further confirmations, and observed quantitatively the same behavior. Note that it is practically hard to perform any serious disorder averaging for the present very large system size.

## III. SIMULATION RESULTS AND DISCUSSIONS

### A. Finite-temperature phase-coherence transition

First, we study the phase-coherence transition. The current-voltage characteristics are simulated at various temperatures ranged from  $[0.2, 0.6]$ . At each temperature, we try to probe the system at a current as low as possible. The voltage is determined when a steady state is reached. Figure 1 shows the resistivity  $R = V/I$  as a function of current  $I$  at various temperatures. It is clear that, at lower temperatures,  $R$  tends to zero as the current decreases, which follows that there is a true superconducting phase with zero linear resis-

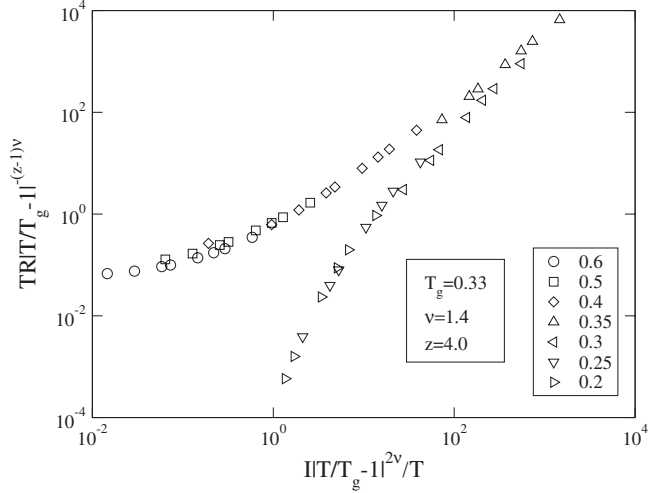


FIG. 2. Dynamic scaling of current-resistivity data at various temperatures according to Eq. (4).

tivity. While  $R$  tends to a finite value at higher temperatures, corresponding to an Ohmic resistivity. These observations provide a strong evidence of occurrence of a phase-coherence transition at finite temperature in the 3D XY spin glass in the dynamical sense.

For a continuous phase transition characterized by the divergence of the characteristic length and time scales  $t \sim \xi^z$  ( $z$  is the dynamic exponent), Fisher *et al.*<sup>28</sup> proposed the following dynamic scaling ansatz:

$$TR\xi^{z+2-d} = \Psi_{\pm}(I\xi^{d-1}/T), \quad (4)$$

where  $d$  is the dimension of the system ( $d=3$  in this paper), and  $\xi \propto |T/T_g - 1|^{-\nu}$  is the correlation length that diverges at the transition.  $\Psi(x)$  is a scaling function, with  $+$  and  $-$  signs corresponding to  $T > T_g$  and  $T < T_g$ . Equation (4) was often used to scale measured current-voltage data experimentally.<sup>30,31</sup>

To extract the critical behavior from the numerical results of the current-voltage characteristics, we will also perform a dynamical scaling analysis. As shown in Fig. 2, using  $T_g = 0.33 \pm 0.02$ ,  $z = 4.0 \pm 0.1$ , and  $\nu = 1.4 \pm 0.1$ , an excellent collapse is achieved according to Eq. (4). The value of  $\nu$  is by no means close to the Ising value  $2.15 \pm 0.15$ ,<sup>11</sup> demonstrating the XY and Ising spin glass belong to different universal classes.

The finite-size effect is particularly significant at temperatures sufficiently close to  $T_g$  when the correlation length exceeds the system size. For the temperatures considered and the very lattice size  $L=64$  here, we believe that the finite-size effect is negligible in the present simulations. To confirm this point, we perform particular simulations right at  $T_g=0.33$  obtained above for different system size. At  $T_g$ , the correlation length is cut off by the system size in any finite system, the scaling form (4) for  $d=3$  becomes

$$T_g R L^{z-1} = \Psi(IL^2/T_g). \quad (5)$$

As shown in Fig. 3, a good collapse is shown using  $z = 4.2 \pm 0.1$ . This consistence demonstrates that the estimate from Fig. 2 is reliable. Therefore, evidence of a finite-

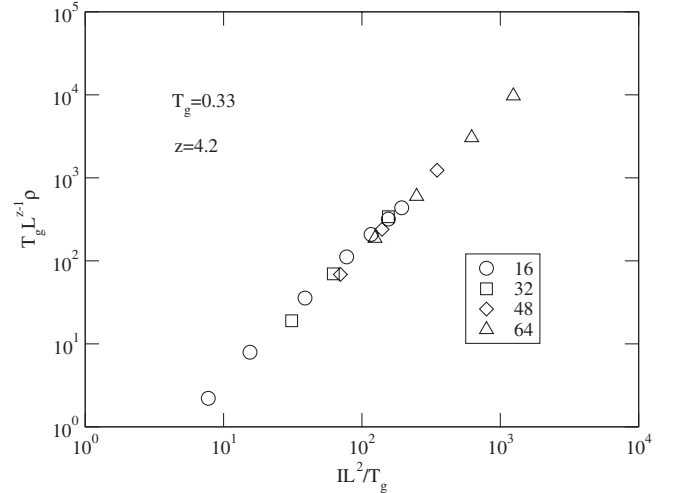


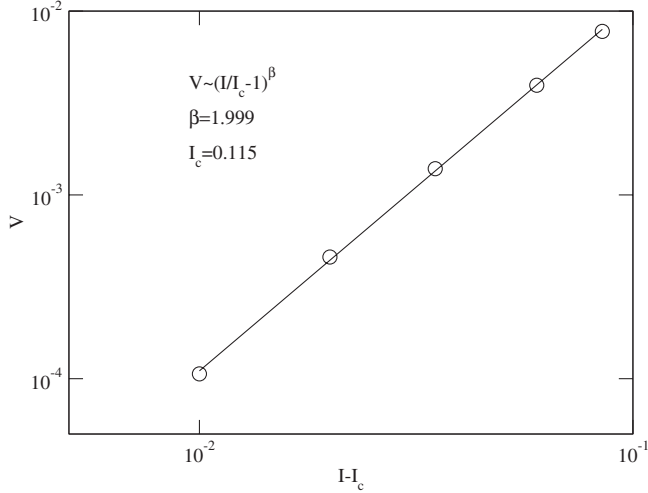
FIG. 3. Dynamic scaling of current-resistivity data at  $T_g=0.33$  according to Eq. (5).

temperature phase-coherence transition is provided convincingly in the 3D XY spin glass.

The above obtained  $T_g$  and static exponent  $\nu$  are consistent with  $T_g = 0.34 \pm 0.02$  and  $\nu = 1.2 \pm 0.2$  obtained in equilibrium Monte Carlo simulations<sup>8</sup> for lattice sizes up to  $L=12$ . More recently, the data for the spin-glass and chiral-glass correlation length at larger sizes show marginal behavior, indicating that the lower critical dimension is close to 3 for both spins and chiralities.<sup>19</sup> The crossing temperatures for the spin and chiral-glass correlation length decrease very slightly with the size with large uncertainties, close to 0.3 for largest lattice sizes they can simulate. The data for the ratio of the chiral and spin-glass correlation for the largest sizes intersects for  $T$  about 0.33 and then splay out in the low temperature side. The size dependence of the crossing temperature at large sizes is very weak, supporting a single transition for both spins and chiralities. Interestingly, this crossing temperature is in excellent agreement with the present  $T_g$  for a large lattice size. We do not think it is a coincidence.

Previous driven Monte Carlo dynamical simulation on the same model for a small lattice size ( $L=12$ ) (Ref. 9) estimated  $T_g = 0.335$ ,  $\nu = 1.2$ , and  $z = 4.4$ , basically consistent with the present ones. It follows that the results are not sensitive to the system size in the dynamical simulations, even the detailed dynamics is different. The Monte Carlo dynamical simulations in the vortex representation also show an equilibrium transitive transition.<sup>12</sup> The estimated value for  $\nu = 1.3 \pm 0.3$  agrees with the present one. But the dynamic exponent  $z \approx 3.1$  is obvious lower, indicating different dynamic universality class for the XY spin glass in the phase and vortex representations.

Typical values of the correlation length exponent and the dynamical exponent extracted from the various experiments on high- $T_c$  cuprate superconductors<sup>30,31</sup> fall in the range of  $\nu \approx 1.0-1.7$  and  $z \approx 4.0-5.0$ . Our results for these static and dynamic exponents in the XY spin glass are consistent with these experimental ones. In addition, nonlinear resistivity measurement in the  $\text{YBa}_2\text{Cu}_3\text{O}_8$  bulk sample<sup>24</sup> near the onset of the paramagnetic Meissner effect have been interpreted as

FIG. 4. Log-log plots of  $V$ - $I$  curve at zero temperatures.

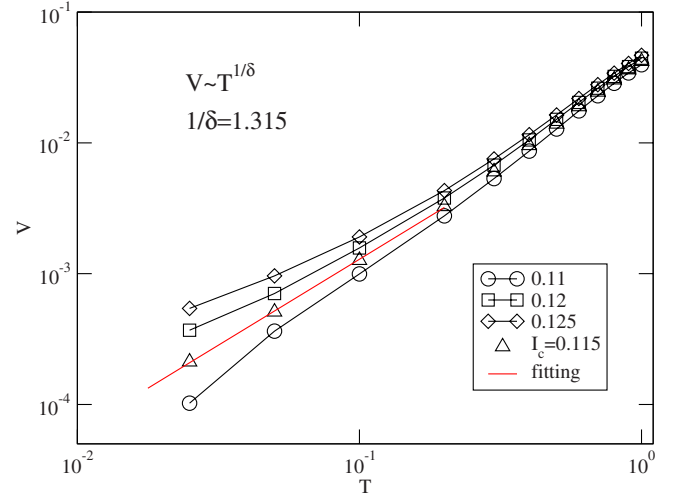
a chiral-glass transition owing to the presence of  $\pi$  junctions. The nonlinear contribution  $\rho_2$  to the resistivity was observed to have a peak at the transition with power-law behavior  $\rho \sim J^{-\alpha}$  and exponent  $\alpha = 1.1 \pm 0.6$ , which has been reproduced by a generalized XY spin-glass model.<sup>26</sup> Granato proposed that it could also be interpreted as a consequence of the underlining phase coherence transition and the exponent  $\alpha$  is determined by the dynamic exponent through  $\alpha = (5 - z)/2$ .<sup>9</sup> By the above obtained dynamic exponent, we have  $\alpha = 0.5 \pm 0.05$ , compatible with the experimental value within the error bar. Note that the present value of  $z$  in the large-scale simulations is smaller than those previously given by Granato<sup>9,10</sup> by about 10%, so our estimated  $\alpha$  is more close to experimental one. Based on the above comparisons with experiments, we propose that the XY spin glass gives an effective description of the transport properties near the phase-coherence transition in  $d$ -wave superconductors.

Until now, at least there is a general consensus in 3D XY spin glass that the chiral-glass transition occurs at a finite temperature larger than or equal to the spin-glass transition temperature  $T_g$ .<sup>4-10,19</sup> The data for the chiral correlation length in the recent large-scale equilibrium Monte Carlo simulations<sup>19</sup> reveals that the chiral-glass transition temperature is around 0.30–0.33, not larger than the present spin-glass transition temperature. We therefore suggest that the spin and the chirality probably order simultaneously.

### B. Depinning transition and creep motion

With the low temperature spin-glass phase in hand, in the remaining part of the paper, we will study the depinning and creep phenomena in this phase.

In the zero-temperature depinning phenomena, it is expected that the voltage should be finite above the threshold (critical) current  $I_c$  and become zero below  $I_c$ . To study the depinning transition, we start from high currents with random initial phase configurations. The current is then lowered step by step. Figure 4 shows the current-voltage characteristics at  $T=0$ . Interestingly, we observe continuous depinning transitions with unique depinning currents, which can be de-

FIG. 5. (Color online) Log-log plots of  $V$ - $T$  curves at three currents around  $I_c$ .

scribed as  $V \propto (I - I_c)^\beta$  with  $I_c = 0.115 \pm 0.001$ ,  $\beta = 1.99 \pm 0.02$ . Note that the depinning exponent  $\beta$  is greater than 1, consistent with the mean field studies on charge density wave models.<sup>36</sup>

At low temperatures, even below the critical current, a finite voltage can be generated by creep motion of vortices due to the thermally activation, so the current-voltage characteristics is rounded near the zero-temperature critical current. An obvious crossover between the depinning and creep motion can be observed around  $I_c$  at the lowest accessible temperatures. In order to address the thermal rounding of the depinning transition, Fisher<sup>36</sup> first suggested to map this system to the ferromagnet in fields where the second-order phase transitions occur. This mapping was latter extended to the random-field Ising model,<sup>37</sup> flux lines in type-II superconductors,<sup>33</sup> and Josephson junction arrays.<sup>38</sup> If the voltage is identified as the order parameter, the current and temperature are identified as the inverse temperature and the field in the ferromagnetic system, respectively, analogous to the second-order phase transitions, a scaling relation among the voltage, current, and temperature reads

$$V(T, I) = T^{1/\delta} S[T^{-1/\beta\delta}(1 - I_c/I)], \quad (6)$$

where  $S(x)$  is a scaling function.

Equation (6) reveals that right at  $I_c$  the voltage shows a power-law behavior  $V(T, I = I_c) \propto T^{1/\delta}$ , providing a method to determine the critical exponent  $1/\delta$ . The  $V$ - $T$  curves at three currents are presented in Fig. 5 on a log-log plot. The critical current is seen to be between 0.11 and 0.125. The values of voltage at other currents within (0.11, 0.125) can be evaluated by quadratic interpolation. The square deviations from the power law can be calculated. The critical current is defined at which the square deviation is minimal. We obtain  $I_c = 0.115 \pm 0.02$ , consistent with those obtained above at zero temperature. The temperature dependence of voltage at the critical current is also plotted in Fig. 5, yielding  $1/\delta = 1.315 \pm 0.001$ .



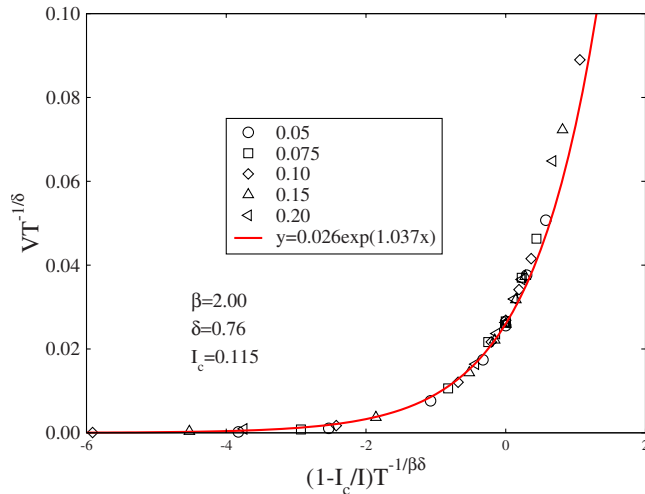


FIG. 6. (Color online) Scaling plot of the current-voltage data at various temperatures below  $T_g$  according to Eq. (6).

With the critical exponent  $\delta$  and the critical current  $I_c$ , we can adjust the depinning exponent  $\beta$  to achieve the best data collapse according to the scaling relation Eq. (6) for  $I \leq I_c$ . In Fig. 6, a optimal data collapse of the current-voltage data at various temperatures below  $T_g$  provides an estimate of  $\beta = 2.00 \pm 0.01$ , which is in excellent agreement with those derived at  $T=0$  depinning transition. Moreover, the scaling function with the form  $V \propto T^{1/\delta} \exp[A(1 - \frac{I_c}{I})/T^{\beta\delta}]$  is used to fit well the current-voltage data in the creep regime, which is also list in the legends of Fig. 6. As we know, the product of the two exponents  $\beta\delta$  describes the temperature dependence of the creeping law.  $\beta\delta \approx 1.52$  deviates from unity, demonstrating that the creep law is a non-Arrhenius type. The non-Arrhenius type creep behaviors have been also previously observed in charge density waves.<sup>39</sup>

In the 3D XY spin glass, the existence of a stable spin-glass phase at finite temperature is well-established through both previous equilibrium studies<sup>8,19</sup> and present dynamical simulations. The value of  $\beta\delta$  is close to 3/2, similar to the 3D vortex glass and gauge glass model.<sup>34</sup> We believe that the value of  $\beta\delta$  is generally 3/2 in the glass phases in various strongly disordered 3D frustrated XY model. The further analytical work is highly called for.

#### IV. CONCLUSIONS

We have performed large-scale dynamical simulations on the 3D XY spin glass within the resistively shunted-junction dynamics. The strong evidence for the low temperature spin-glass phase is provided in dynamical sense. By the dynamical scaling analysis, a perfect collapse of simulated current-voltage data is achieved by using  $T_g = 0.33 \pm 0.02$ ,  $z = 4.0 \pm 0.1$ , and  $\nu = 1.4 \pm 0.1$ . These critical values agree with those in the previous equilibrium Monte Carlo simulations as well as driven Monte Carlo dynamical simulations. In the resistive simulations, the size dependence of various exponents and critical temperature in the phase-coherence transition are very weak at larger size, providing a good technique to measure critical properties in this model. The static and dynamic exponents are compatible with those observed in the various experiments on high- $T_c$  cuprate superconductors, which underlines the significance of XY spin glass in  $d$ -wave superconductors. Combined with the recent equilibrium Monte Carlo simulations, we suggest a single glass transition involving both spins and chiralities.

We also study the depinning transition at zero temperature and creep motion at low temperatures in detail. A genuine continuous depinning transition is observed and the depinning exponent is evaluated. With the notion of scaling, the critical exponents are estimated, which are consistent with those from independent simulations at zero temperature and at critical current. The value of  $\beta\delta$  is close to 3/2 and the scaling curve is fitted well by a exponential function, suggesting a non-Arrhenius type creep motion in the low temperature ordered phase in the XY spin glass.

Finally, it is proposed that the XY spin-glass model may capture the essential transport feature in high- $T_c$  cuprate superconductors with  $d$ -wave symmetry. Further experimental and theoretical works are clearly motivated.

#### ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China under Grant No. 10774128, PCSIRT (Grant No. IRT0754) in University in China, National Basic Research Program of China (Grants No. 2006CB601003 and No. 2009CB929104), Zhejiang Provincial Natural Science Foundation under Grant No. Z7080203, and the Program for Innovative Research Team in Zhejiang Normal University.

\*qhchen@zju.edu.cn

<sup>1</sup>S. Jain and A. P. Young, J. Phys. C **19**, 3913 (1986); J. A. Olive, A. P. Young, and D. Sherrington, Phys. Rev. B **34**, 6341 (1986).

<sup>2</sup>J. Villain, J. Phys. C **10**, 4793 (1977).

<sup>3</sup>H. Kawamura and M. Tanemura, Phys. Rev. B **36**, 7177 (1987).

<sup>4</sup>H. Kawamura and M. S. Li, Phys. Rev. Lett. **87**, 187204 (2001).

<sup>5</sup>J. Maucourt and D. R. Grempel, Phys. Rev. Lett. **80**, 770 (1998).

<sup>6</sup>N. Akino and J. M. Kosterlitz, Phys. Rev. B **66**, 054536 (2002).

<sup>7</sup>F. Matsubara, T. Shirakura, and S. Endoh, Phys. Rev. B **64**, 092412 (2001); S. Endoh, F. Matsubara, and T. Shirakura, J. Phys. Soc. Jpn. **70**, 1543 (2001); T. Nakamura and S. Endoh,

*ibid.* **71**, 2113 (2002); T. Yamamoto, T. Sugashima, and T. Nakamura, Phys. Rev. B **70**, 184417 (2004).

<sup>8</sup>L. W. Lee and A. P. Young, Phys. Rev. Lett. **90**, 227203 (2003).

<sup>9</sup>E. Granato, Phys. Rev. B **69**, 144203 (2004).

<sup>10</sup>E. Granato, Phys. Rev. B **69**, 012503 (2004).

<sup>11</sup>H. G. Ballesteros, A. Cruz, L. A. Fernandez, V. Martin-Mayor, J. Pech, J. J. Ruiz-Lorenzo, A. Tarancon, P. Tellez, C. L. Ullod, and C. Ungil, Phys. Rev. B **62**, 14237 (2000).

<sup>12</sup>C. Wengel and A. P. Young, Phys. Rev. B **56**, 5918 (1997).

<sup>13</sup>A. P. Young, J. Phys.: Conf. Ser. **95**, 012003 (2008).

<sup>14</sup>K. Hukushima and H. Kawamura, Phys. Rev. B **72**, 144416

- (2005).
- <sup>15</sup>I. Campos, M. Cotallo-Aban, V. Martin-Mayor, S. Perez-Gaviro, and A. Tarancon, Phys. Rev. Lett. **97**, 217204 (2006).
  - <sup>16</sup>L. W. Lee and A. P. Young, Phys. Rev. B **76**, 024405 (2007).
  - <sup>17</sup>D. X. Viet and H. Kawamura, Phys. Rev. Lett. **102**, 027202 (2009); Phys. Rev. B **80**, 064418 (2009).
  - <sup>18</sup>L. A. Fernandez, V. Martin-Mayor, S. Perez-Gaviro, A. Tarancon, and A. P. Young, Phys. Rev. B **80**, 024422 (2009).
  - <sup>19</sup>J. H. Pixley and A. P. Young, Phys. Rev. B **78**, 014419 (2008).
  - <sup>20</sup>R. Mathieu, A. Asamitsu, Y. Kaneko, J. P. He, and Y. Tokura, Phys. Rev. B **72**, 014436 (2005).
  - <sup>21</sup>M. Sigrist and T. M. Rice, Rev. Mod. Phys. **67**, 503 (1995).
  - <sup>22</sup>H. Kawamura, J. Phys. Soc. Jpn. **64**, 711 (1995).
  - <sup>23</sup>M. Matsuura, M. Kawachi, K. Miyoshi, M. Hagiwara, and K. Koyama, J. Phys. Soc. Jpn. **64**, 4540 (1995).
  - <sup>24</sup>T. Yamao, M. Hagiwara, K. Koyama, and M. Matsuura, J. Phys. Soc. Jpn. **68**, 871 (1999).
  - <sup>25</sup>E. L. Papadopolou, P. Nordblad, P. Svedlindh, R. Schöneberger, and R. Gross, Phys. Rev. Lett. **82**, 173 (1999); E. L. Papadopolou, P. Nordblad, and P. Svedlindh, Physica C **341-348**, 1379 (2000).
  - <sup>26</sup>M. S. Li and D. Domínguez, Phys. Rev. B **62**, 14554 (2000).
  - <sup>27</sup>M. P. A. Fisher, Phys. Rev. Lett. **62**, 1415 (1989).
  - <sup>28</sup>D. S. Fisher, M. P. A. Fisher, and D. A. Huse, Phys. Rev. B **43**, 130 (1991).
  - <sup>29</sup>T. Nattermann and S. Scheidl, Adv. Phys. **49**, 607 (2000).
  - <sup>30</sup>R. H. Koch, V. Foglietti, W. J. Gallagher, G. Koren, A. Gupta, and M. P. A. Fisher, Phys. Rev. Lett. **63**, 1511 (1989); R. H. Koch, V. Foglietti, and M. P. A. Fisher, *ibid.* **64**, 2586 (1990); P. L. Gammel, L. F. Schneemeyer, and D. J. Bishop, *ibid.* **66**, 953 (1991).
  - <sup>31</sup>T. Klein, A. Conde-Gallardo, J. Marcus, C. Escribe-Filippini, P. Samuely, P. Szabo, and A. G. M. Jansen, Phys. Rev. B **58**, 12411 (1998); A. M. Petrean, L. M. Paulius, W.-K. Kwok, J. A. Fendrich, and G. W. Crabtree, Phys. Rev. Lett. **84**, 5852 (2000).
  - <sup>32</sup>Q. H. Chen and X. Hu, Phys. Rev. Lett. **90**, 117005 (2003).
  - <sup>33</sup>M. B. Luo and X. Hu, Phys. Rev. Lett. **98**, 267002 (2007).
  - <sup>34</sup>Q. H. Chen, Phys. Rev. B **78**, 104501 (2008); EPL **84**, 64001 (2008).
  - <sup>35</sup>A. I. Larkin and Yu. N. Ovchinnikov, J. Low Temp. Phys. **34**, 409 (1979); L. B. Ioffe and V. M. Vinokur, J. Phys. C **20**, 6149 (1987); T. Nattermann, EPL **4**, 1241 (1987); M. V. Feigel'man, V. B. Geshkenbein, A. I. Larkin, and V. M. Vinokur, Phys. Rev. Lett. **63**, 2303 (1989); P. Chauve, T. Giamarchi, and P. Le Doussal, Phys. Rev. B **62**, 6241 (2000).
  - <sup>36</sup>D. S. Fisher, Phys. Rev. Lett. **50**, 1486 (1983); Phys. Rev. B **31**, 1396 (1985).
  - <sup>37</sup>L. Roters, A. Hucht, S. Lubeck, U. Nowak, and K. D. Usadel, Phys. Rev. E **60**, 5202 (1999).
  - <sup>38</sup>H. Liu, W. Zhou, and Q. H. Chen, Phys. Rev. B **78**, 054509 (2008).
  - <sup>39</sup>A. A. Middleton, Phys. Rev. B **45**, 9465 (1992).